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The symmetrical properties of quarks and leptons

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ABSTRACT

Symmetry is used to define a particle set is that possesses charges, masses, and couplings that closely coincide with the phenomenology of quarks and leptons. This is achieved by placing the quarks and leptons at the twelve vertices of a cuboctahedron. These are then distinguished by properties that are either 2-valued, 3-valued, 4-valued, or 6-valued, which correspond to nature's four forces: the strong, weak, electromagnetic, and gravitational, respectively. So, a 2-valued symmetry differentiates quarks from leptons; a 3-valued symmetry determines their generations (and thereby their dominant couplings); a 4-valued symmetry produces their electric charges; and a 6-valued symmetry helps generate eight key mass ratios with the aid of an empirical mass formula.

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1. INTRODUCTION

Symmetry is used to define a particle set is that possesses charges, masses, and couplings that closely coincide with the phenomenology of quarks and leptons. This is achieved by placing the quarks and leptons at the twelve vertices of a cuboctahedron. These are then distinguished by properties that are either 2-valued, 3-valued, 4-valued, or 6-valued, which correspond to nature's four forces: the strong, weak, electromagnetic, and gravitational, respectively. The results achieved follow those achieved earlier by the author [1].

2. GENERATING QUARK AND LEPTON PROPERTIES

We begin by noting that the vertices of a cuboctahedron form four rings, six squares, and eight triangles. In Figure 1, we label these twelve vertices so as to assign, in a symmetrical way, values for Q, G, and R to each quark and lepton, where:

- $Q = \{ 0, -⅓, +⅔, -1 \}$, the value of electric charge,
- $G = \{ -1, 0, +1 \}$, quark or lepton generation, and,
- $R = \{ 0, 1, 1, 2, 3, 5 \}$, a new quantum number, drawn from the first six Fibonacci numbers, that will play a key role in determining a quark or lepton's mass.

The assignments for G are summarized in Table I and govern weak coupling. Observe that we assume neutrinos to be Majorana (i.e., $\nu_x = \bar{\nu}_x$), which causes some neutrinos to be members of generations $+1$ *and* -1; also observe that a particle in generation G has its antiparticle in $-G$. Finally, we establish the following *Weak Coupling Rule*:

> *Particles may couple via a* $W^{\pm}$ *only if they possess electric charges Q that differ by 1 and generations G that sum to 0.*

Now notice that the cuboctahedron of Figure 1 exhibits the following symmetries:

1) Each of its four rings contains three quarks and three leptons;
2) Each of its six squares has absolute values for Q that sum to 2;
3) Each of its four rings has generations G of $\{ -1, -1, 0, 0, +1, +1 \}$;
4) Each of its eight triangles has values of R that sum to 6;
5) The values for Q in each generation are $\{ 0, -\frac{1}{3}, +\frac{2}{3}, -1 \}$;
6) The values for Q and G allow each particle to weakly couple via a $W^{\pm}$ to just one other particle (except in cases of mixing arising from the assumption that $\nu_x = \bar{\nu}_x$).

As these symmetries make clear, the assignments of the cuboctahedron of Figure 1 are rigidly constrained. It follows that the quark and lepton phenomenology they give rise to—examined immediately below—arises largely from these symmetries.

3. PARTICLE GENERATIONS, COUPLING, AND MIXING

We have already seen that Figure 1 assigns values of charge and generation to all quarks and leptons. Accordingly, we apply the *Weak Coupling Rule* to recover the following couplings

$$
\begin{array}{ccccccc}
b & \longleftrightarrow & t & \qquad & \nu_3 & \longleftrightarrow & \tau \\
s & \longleftrightarrow & c & \qquad & \nu_2 & \longleftrightarrow & \mu \\
d & \longleftrightarrow & u & \qquad & \nu_1 & \longleftrightarrow & \mathrm{e}
\end{array}
$$

which accurately reflect the dominant couplings found by experiment [2].

Note specifically that, above, the mixing between ν_2 and ν_3 arises because they each occupy two generations (+1 and −1), so that the tau and muon couple to both. In this way the large mixing observed between the υ_2 and υ_3 mass eigenstates is accounted for [2].

4. QUARK AND LEPTON MASS RATIOS

4.1 Generating quark and lepton mass ratios

Before we generate the quark and lepton mass ratios, we need first to assign values for T. We let $T = 1$ for the *u*-, *d*-, *s*-, and *c*-quarks (which in the cuboctahedron of Figure 1 reside in a single plane); and let $T = 0$ for all other particles. Furthermore, we let $m = 1$ for heavy particles; and $m = 2$ for light particles.

These assignments, along with values for R taken from Figure 1, allow the following mass ratios

$\frac{M(\tau)}{M(e)}$	$\frac{M(t)}{M(c)}$
$\frac{M(\mu)}{M(e)}$	$\frac{M(b)}{M(c)}$

to be generated by the empirical mass formula

$$M(\Delta R, \Delta T) = 4.1^{\frac{\Delta R}{m}} \times 3^{\frac{1}{m}} \times \frac{1}{10^{\Delta T}} \quad . \tag{1}$$

$m = 1$	
$M(R_\tau - R_e,\ T_\tau - T_e) = 4.1^5 \times 3$	$M(R_t - R_c,\ T_t - T_c) = 4.1 \times 3 \times 10$
$M(R_\mu - R_e,\ T_\mu - T_e) = 4.1^3 \times 3$	$M(R_b - R_c,\ T_b - T_c) = 3$

Likewise, we can generate the following mass ratios for light quarks and leptons

$\frac{M(\nu_3)}{M(\nu_1)}$	$\frac{M(s)}{M(u)}$
$\frac{M(\nu_2)}{M(\nu_1)}$	$\frac{M(d)}{M(u)}$

with these light particle counterparts of the heavy particle equations.

$m = 2$	
$M\left(R_{\nu_3} - R_{\nu_1},\ T_{\nu_3} - T_{\nu_1} \right) = 4.1^{\frac{5}{2}} \times 3^{\frac{1}{2}}$	$M\left(R_s - R_u,\ T_s - T_u \right) = 4.1^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 10$
$M\left(R_{\nu_2} - R_{\nu_1},\ T_{\nu_2} - T_{\nu_1} \right) = 4.1^{\frac{3}{2}} \times 3^{\frac{1}{2}}$	$M\left(R_d - R_u,\ T_d - T_u \right) = 3^{\frac{1}{2}}$

4.2 Comparison against experiment

The calculated values for the heavy quark mass ratios fit their experimental values within, or very close to, their limits of error, while the various ratios associated with the light quarks are well within their limits of error [2]. In addition, the calculated values for $\frac{M(\tau)}{M(e)}$ and $\frac{M(\mu)}{M(e)}$ fit their corresponding experimental values unusually closely: to about 1 part in 2,000, and 1 part in 40,000, respectively [2]. For neutrinos it is possible to calculate the ratio of two squared-mass splittings, where its corresponding experimental value proves to be too large by a factor of about two [2,3].

4.3 Degrees of freedom

It is logical to ask whether *any* experimental mass data might be fit by a mass formula whose constants are adjusted with sufficient care. As it is, the constants 3, $\frac{41}{10}$, and $\frac{1}{10}$ of Eq. (1) are needed simply to fit $\frac{M(b)}{M(c)}$, $\frac{M(\mu)}{M(e)}$, and $\frac{M(t)}{M(c)}$, respectively. It follows that the fit of $\frac{M(\tau)}{M(e)}$ to

1 part in 2,000 arises independently, and therefore offers key evidence of the mass formula's physical significance. Moreover, it is not trivial that the cuboctahedron's symmetrical assignments give rise to masses, charges, and couplings that are correctly joined in all instances.

5. CONCLUSION

It is striking that the above symmetries, which are 2-, 3-, 4-, and 6-valued, neatly correspond to the four known forces. Specifically:

- The 2-valued symmetry is associated with the strong force (in the sense that quarks—but not leptons—feel this force).
- The 3-valued symmetry is associated with particle generation and the weak force.
- The 4-valued symmetry is associated with charge and electromagnetic force.
- The 6-valued symmetry is associated with mass and gravitational force.

Considering the simplicity of these symmetries, and the way they seamlessly unite, it is remarkable that the particle set they generate possesses charges, masses, and couplings that so faithfully coincide with the phenomenology of the quarks and leptons.

REFERENCES

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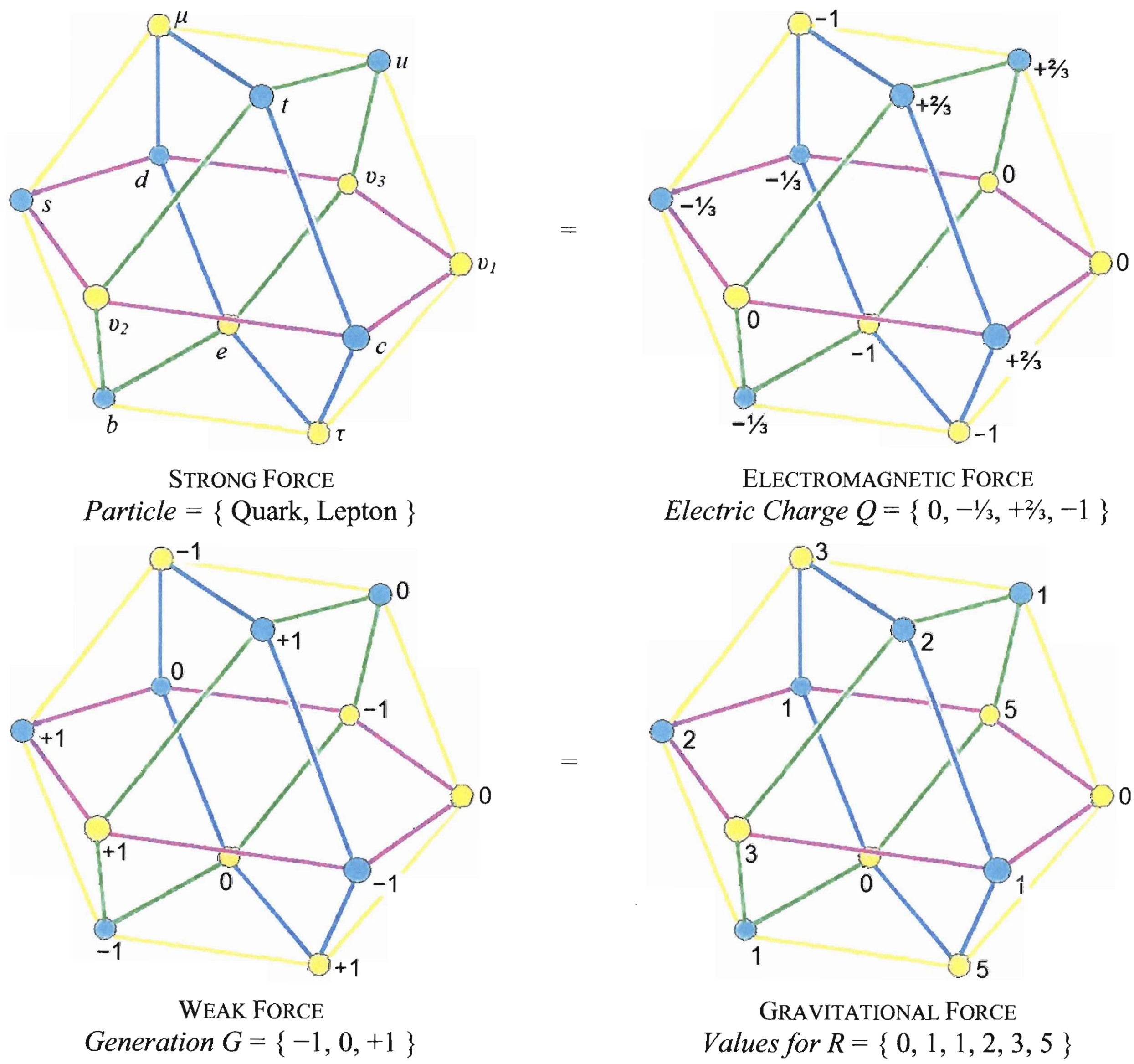

Figure 1. Assignment of values for Q, G, and R to each quark and lepton. Note that, above, the cuboctahedron's vertices define four rings, with each ring containing three quarks and three leptons (at the upper left), and each containing generations −1, −1, 0, 0, +1, +1 (at the lower left). Its surface also consists of six squares, each containing absolute values of Q that sum to 2 (at the upper right), and eight triangles, each containing values of R that sum to 6 (at the lower right).

Table I. A summary of the generational assignments of Figure 1. Particles may couple via a W$^{\pm}$ only if they possess electric charges Q that differ by 1 and generations G that sum to 0. Note that because we assume $\nu_x = \bar{\nu}_x$, some neutrinos are members of both the +1 and −1 generations. These instances, which appear shaded below, give rise to mixing.

Quarks	*Particle*	u	d	s	c	b	t
	Generation	0	0	+1	−1	−1	+1
	Antiparticle	$\bar{u}$	$\bar{d}$	$\bar{s}$	$\bar{c}$	$\bar{b}$	$\bar{t}$
	Generation	0	0	−1	+1	+1	−1
Leptons	*Particle*	ν_1	e^-	ν_2	μ^-	ν_3	τ^-
	Generation	0	0	+1	−1	−1	+1
	Antiparticle	$\bar{\nu}_1$	e^+	$\bar{\nu}_2$	μ^+	$\bar{\nu}_3$	τ^+
	Generation	0	0	−1	+1	+1	−1