APRI-PH-2006-47a
1/19/2006

The symmetrical generation of quark and lepton, mass and charge

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Abstract

The symmetries of a cube are exploited to economically generate the masses and charges of the quarks and leptons. The mass formula employed succeeds in part by exploiting the first six Fibonacci numbers.

I. Introduction

This article details a method for economically generating the masses and charges of the quarks and leptons, with the aid of the symmetries of a cube. We begin by letting A = 3 and B = 1/10 for the mass formula

$$M^{+} = \left(A + B + 1\right)^{\frac{R^{+}}{m}} \times A^{\frac{S}{m}} \times B^{T} \quad , \tag{1}$$

where M^{+} is defined as relative mass. This use of 3 and 1/10 closely follows [1].

The above formula, in combination with the parameters of Table I, will generate the eight independent mass ratios that hold within these four particle subgroups: the heavy quarks, heavy leptons, light quarks, and the neutrino mass eigenstates. These results, which appear in Tables IIa and IIb, also follow [1] and are consistent with those achieved earlier by the author [2,3,4,5,6].

II. Mass Formula Parameter Generation

We begin by showing how the parameters of Table I may be generated by a series of well-defined steps. These steps are described in Figures 1, 2, 3, 4a, 4b, 5a, and 5b.

We begin with Figure 1. This diagram displays the values of m, which for all heavy particles equals 1, and which for all light particles equals 2. In this figure, all light particles appear in the lower ring, while all heavy particles appear in the three remaining rings. The diagram produced is symmetrical about its *vertical axis*.

The diagram of Figure 2 is produced by placing precisely three quarks (Q) and three leptons (L) in each ring, in such a way that they are symmetrical about the diagram's dual *diagonal* axes.

The diagram of Figure 3 is produced by combining the conflicting symmetries (*vertical* and *diagonal*, respectively) of Figures 1 and 2, so that light quarks and leptons in the lower ring of Figure 3 appear in italicized lower case, while heavy quarks and leptons in the remaining rings all appear in uppercase. Note that although generated from Figures 1 and 2, Figure 3 has neither the vertical and diagonal symmetry of these figures.

The diagram of Figure 4a assigns the values of electric charge Q for all quarks and leptons, where the values assumed by Q are { 0, 1, 2, 3 } repeated three times. It is produced from Figure 3 by substituting a charge of 0 for the light leptons (l), and 3 for the heavy leptons (L), and either 1 or 2 for the heavy and light quarks (Q and q, respectively). Importantly, these 1s and 2s must be arranged so that, in groups of *four*, they sum to 6 in *nine* different ways, as is seen in Figure 4b. Note that if the midpoints of the 12 edges of a cube are similarly labeled, these nine sums would correlate with nine orthogonal "slices" of the cube, where each slice contains four such midpoints [1].

The diagram of Figure 5a assigns the values of of R^+ for all quarks and leptons. The values assumed by R^+ are { **0**, **1**, 1, 2, 3, 5 }, which are the first six Fibonacci numbers. These values are repeated twice. These values are arranged so that they are symmetrical about their diagonal axes, and so that, in groups of *three*, they sum to 6 in *eight* different ways, as is seen in Figure 5b. In this way, the 6-valued parameter R^+ is directly analogous to the 4-valued Q.

Note that the Fibonacci sequence extended in the reverse of its normal direction R^- = { −3, 2, −1, 1, **0**, **1** } can serve equally well in reproducing the particle masses [1,2]. Because this dual solution cannot be extended to encompass 8 or more Fibonacci terms, it appears to impose a natural limit of 3 on the number of particle families [2].

Figure 6 summarizes the parameter assignments of Figures 1, 4a, and 5a, but with the diagrams for m, Q and R^{+} rotated 45° clockwise. In addition, the values for the parameters S and T are assigned in the upper left-hand corner of Figure 6. All of the parameters of Figure 6 are summarized in Table I. The parameters of Figure 6 (or, equally, Table I), along the mass formula Eq. (1), are all that is needed to generate the mass ratios of Tables IIa and IIb.

III. Conclusion

As noted earlier, the above generation of the quark and lepton mass ratios matches the results achieved earlier work by the author [1,2], and therefore the issue of its effectiveness need not be revisited here. It is sufficient to note that the above method of parameter generation represents an additional step towards the rigorous axiomatization of the generation of the masses and charges of the quarks and leptons.

References

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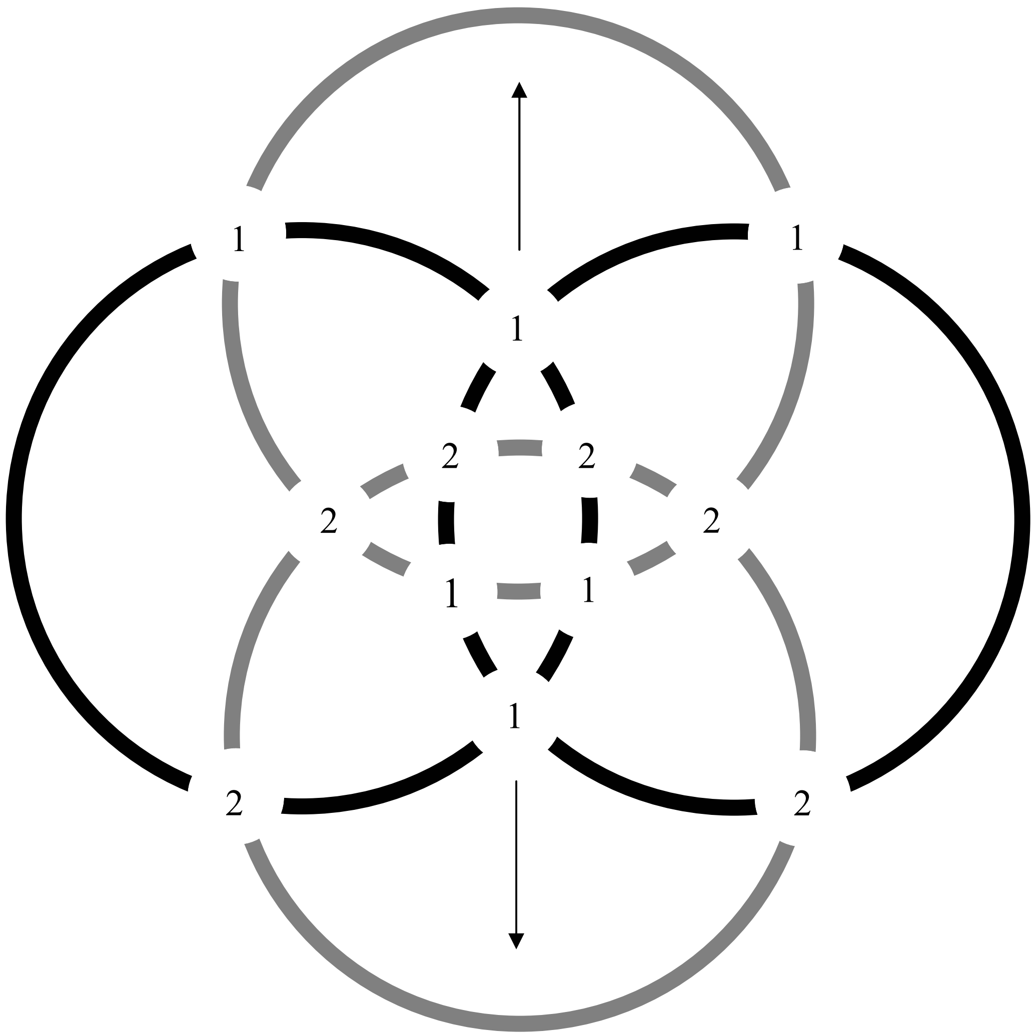

Figure 1. This diagram displays the values of *m*, which for all heavy particles equals 1, and which for all light particles equals 2. The above diagram is produced by locating all light particles (2s) in the lower ring, and all heavy particles (1s) in the remaining rings. The diagram produced is symmetrical about its vertical axis.

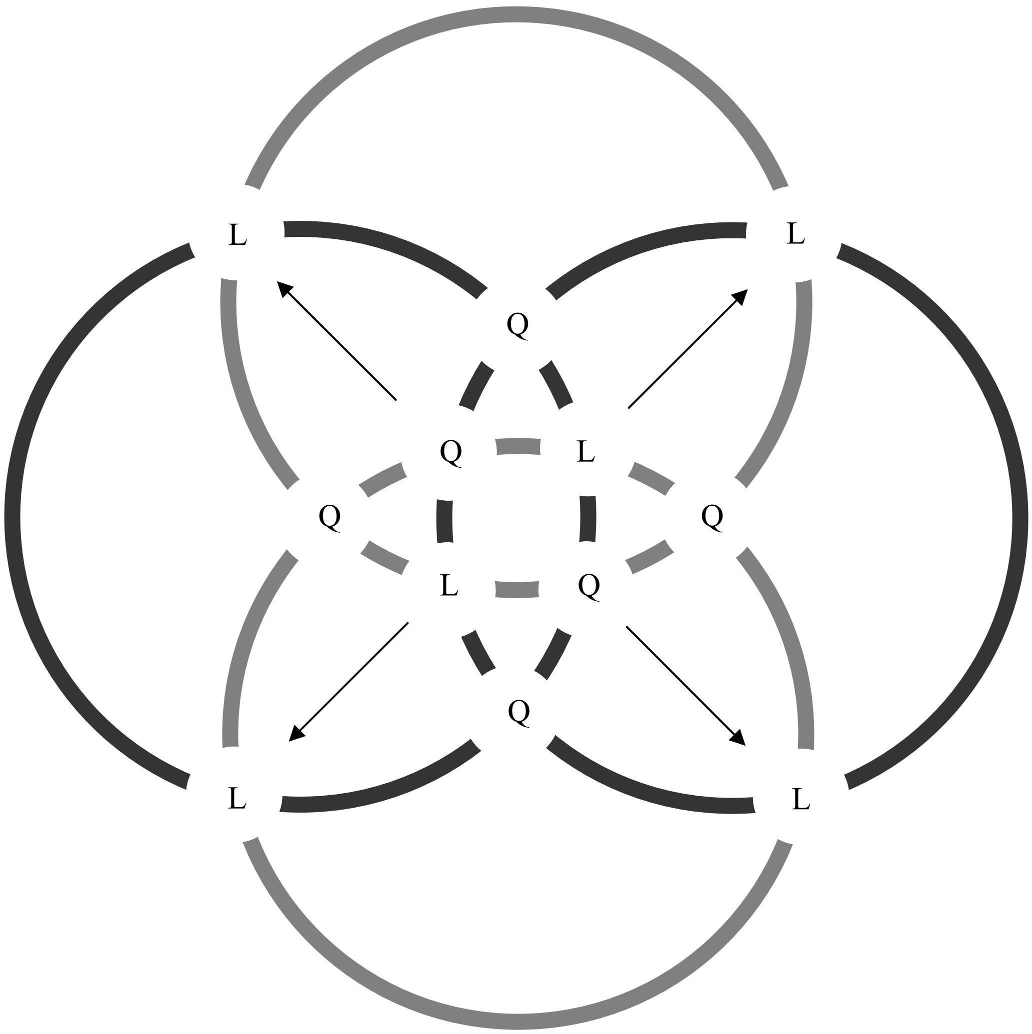

Figure 2. The above diagram is produced by placing precisely three quarks (Q) and three leptons (L) in each ring, so that they are symmetrical about the diagram's diagonal axes.

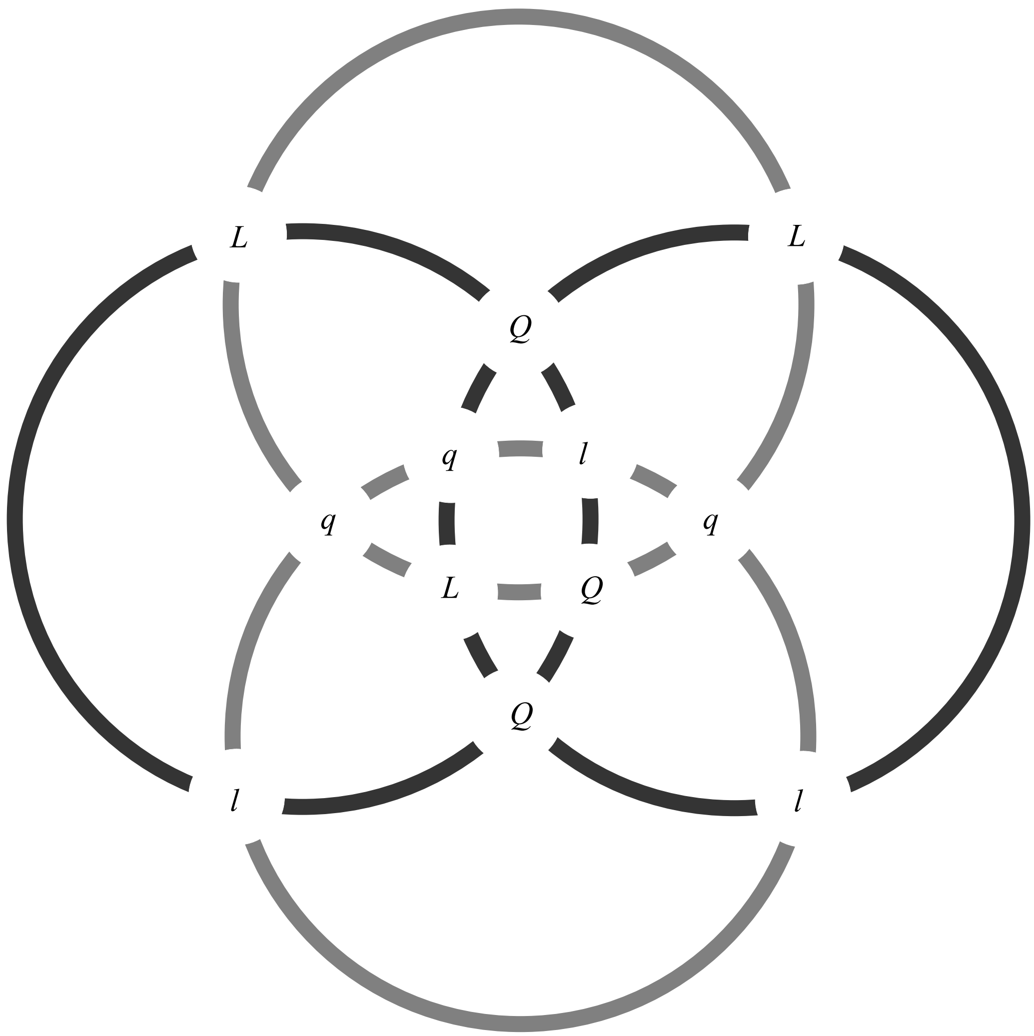

Figure 3. The above diagram is produced from Figure 2 by putting its lower ring in italicized lower case, and its remaining rings in italicized uppercase. In what follows, *q* and *l* will represent light quarks and leptons, respectively, while *Q* and *L* will represent heavy quarks and leptons, respectively. A glance at Figure 1 shows that, as the lower ring of Figure 1 has all 2s, this leaves all light particles with $m = 2$, and all heavy particles with $m = 1$ (see Table I.)

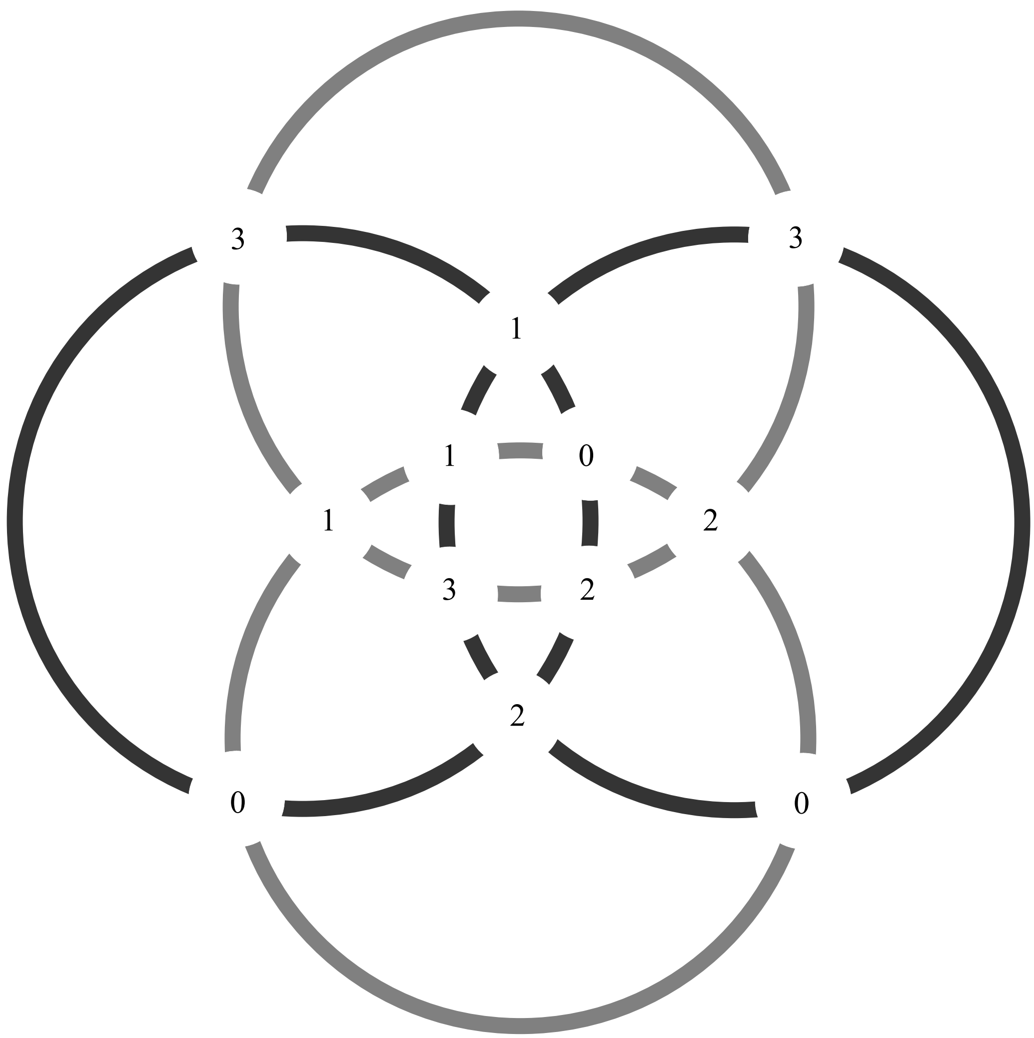

Figure 4a. The above diagram assigns the values of electric charge *Q* for all quarks and leptons. It is produced from Figure 3 by substituting a charge of 0 for the light leptons (*l*), and 3 for the heavy leptons (*L*), and either 1 or 2 for the remaining particles, the light and heavy quarks (*q* and *Q*). These 1s and 2s are arranged so that, in groups of four, they sum to 6 in nine different ways (see Figure 4b).

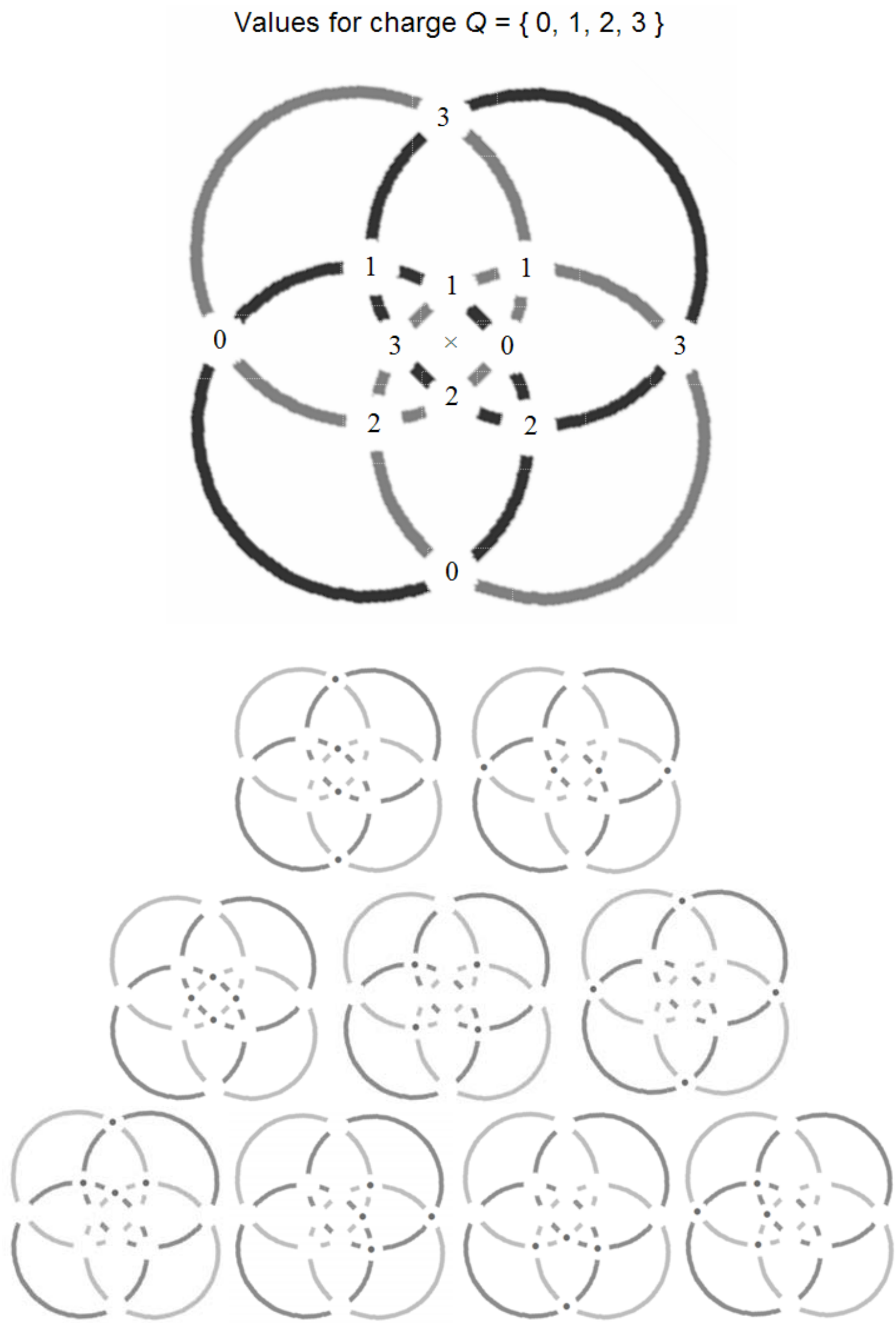

Figure 4b. In the upper diagram, which is Figure 4a rotated 45° clockwise, the values assumed by electric charge are $Q = \{ 0, 1, 2, 3 \}$. These values are repeated three times. The above nine masks group the values of Q by four, so that they sum to 6 in nine different ways. Note that if the midpoints of the 12 edges of a cube are similarly labeled, the above nine sums would correlate with nine orthogonal "slices" of the cube, where each slice contains four such midpoints.

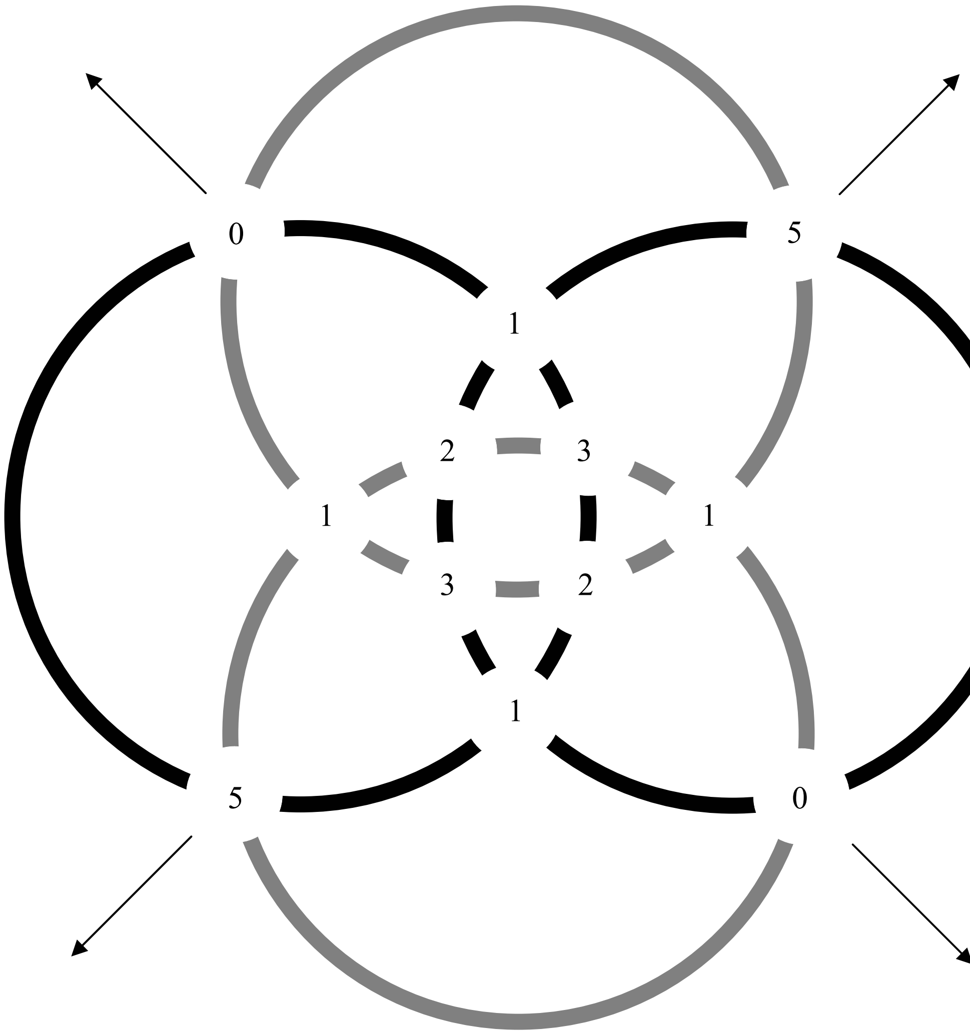

Figure 5a. The above diagram displays the values of R^+ for all quarks and leptons. The values assumed by R^+ are { **0**, **1**, 1, 2, 3, 5 }, which are the first six Fibonacci numbers. These values are repeated twice. These values are arranged so that they are symmetrical about their diagonal axes, and so that, in groups of three, they sum to 6 in eight different ways (see Figure 5b). In this way, the values of R^+ are analogous to those for electric charge Q in Figure 4b. Note that the Fibonacci sequence extended in the reverse of its normal direction $R^- =$ { −3, 2, −1, 1, **0**, **1** } can serve equally well in reproducing the particle masses [1]. Because this dual solution cannot be extended to encompass 8 or more Fibonacci terms, it appears to impose a natural limit of three on the number of particle families [2].

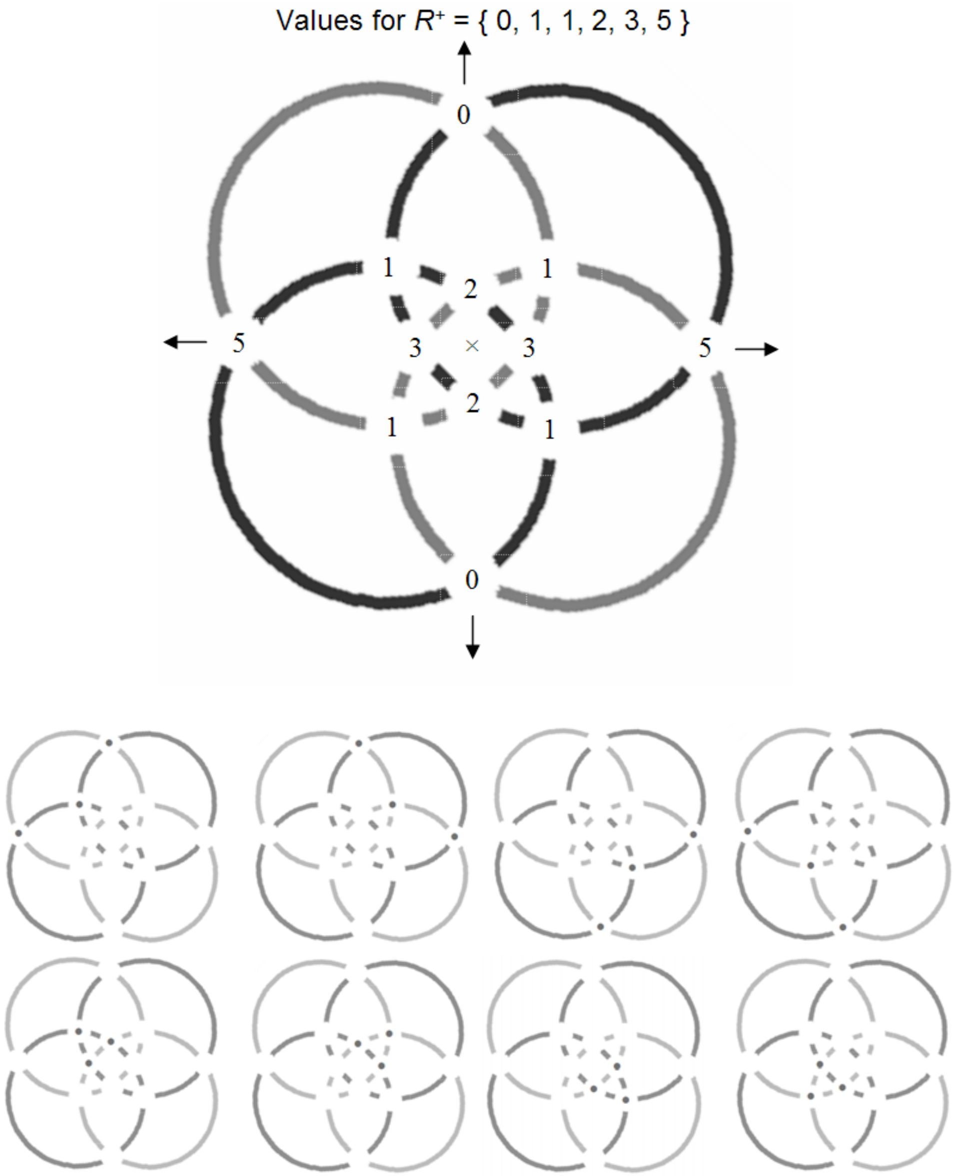

Figure 5b. In the upper diagram, which is Figure 5a rotated 45° clockwise, the values assumed by R^+ are { 0, 1, 1, 2, 3, 5 }, which are the first six Fibonacci numbers. These values are repeated twice. The above eight masks group the values of R^+ by three, so that they sum to 6 in eight different ways. Note that if the midpoints of the 12 edges of a cube are similarly labeled, the above eight sums would correlate with the eight corners of the cube, where each corner involves three such midpoints.

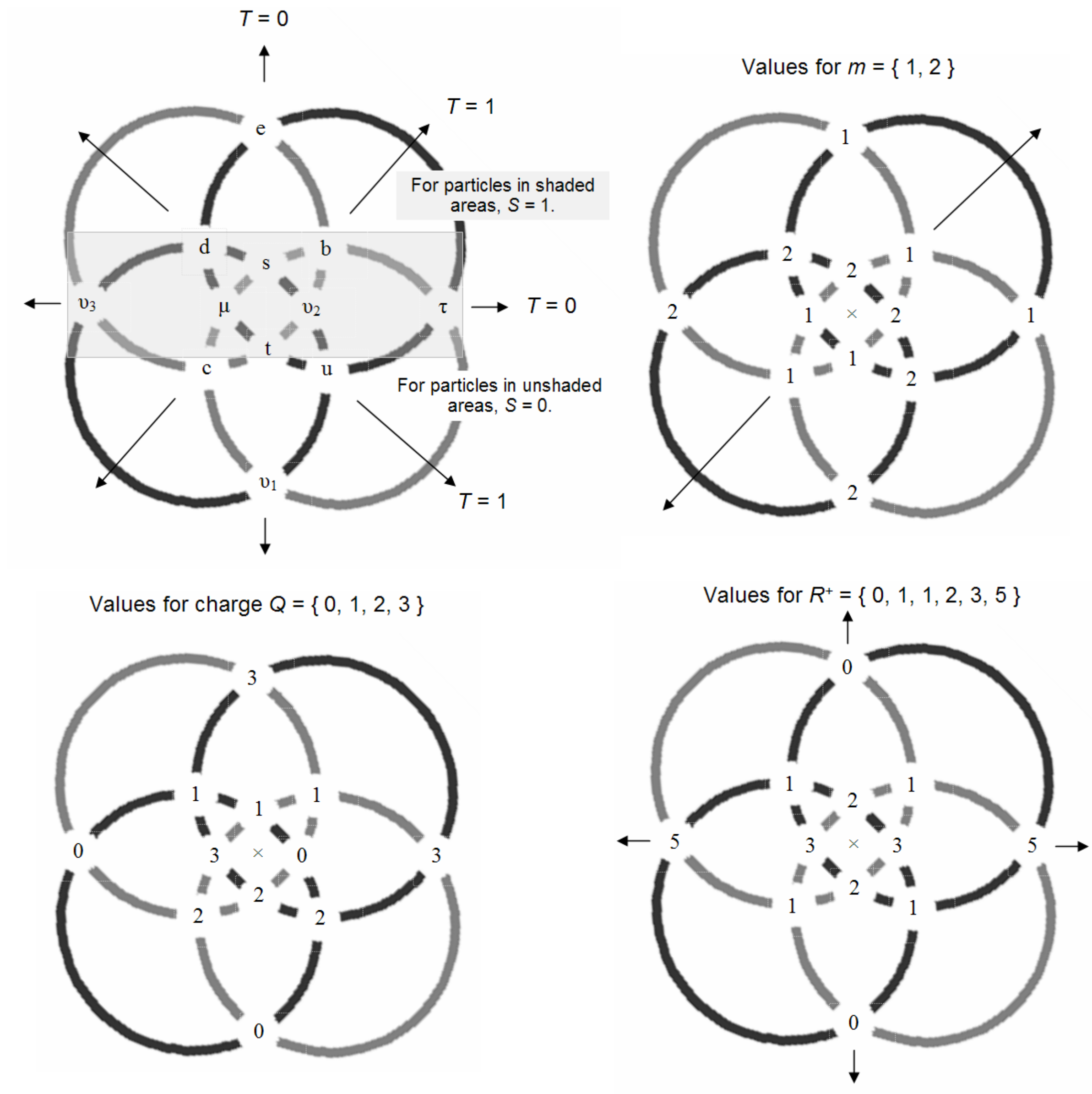

Figure 6. In the above diagrams, the parameters for the mass formula are assigned. The values for S and T are assigned in the upper left-hand corner. Continuing clockwise, we assign the values for m, R^+, and Q. The assignments follow Figures 1, 5a, and 4a, but with the diagrams for m, R^+, and Q rotated 45° clockwise.

Table I. A summary of the parameter assignments of Figure 6. The "symbols" referred to in the first row are those used in Figure 3.

	Heavy Particles						*Light Particles*					
	e	μ	τ	c	b	t	υ_1	υ_2	υ_3	u	d	s
Symbol	*L*	*L*	*L*	*Q*	*Q*	*Q*	*l*	*l*	*l*	*q*	*q*	*q*
Charge Q	3	3	3	2	1	2	0	0	0	2	1	1
R^+	0	3	5	1	1	2	0	3	5	1	1	2
S	0	1	1	0	1	1	0	1	1	0	1	1
T	0	0	0	1	1	0	0	0	0	1	1	0
m	1	1	1	1	1	1	2	2	2	2	2	2

Table IIa. The eight mass ratios produced by mass formula.

Light Particles	*Heavy Particles*	*Mass Ratio*
$\left(\frac{M(\nu_3)}{M(\nu_1)}\right)^2$	$\frac{M(\tau)}{M(e)}$	$4.1^5 \times 3$
$\left(\frac{M(\nu_2)}{M(\nu_1)}\right)^2$	$\frac{M(\mu)}{M(e)}$	$4.1^3 \times 3$
$\left(\frac{0.1 \times M(s)}{M(u)}\right)^2$	$\frac{0.1 \times M(t)}{M(c)}$	$4.1^1 \times 3$
$\left(\frac{M(d)}{M(u)}\right)^2$	$\frac{M(b)}{M(c)}$	$4.1^0 \times 3$

Table IIb. The eight mass ratios produced by mass formula.

Particles	*Mass Ratios*
Heavy leptons	$4.1^5 \times 3 \;:\; 4.1^3 \times 3 \;:\; 1$
Neutrino mass eigenstates	$\sqrt{4.1^5 \times 3} : \sqrt{4.1^3 \times 3} : \sqrt{1}$
Heavy quarks	$4.1 \times 10 \times 3 : 3 : 1$
Light quarks	$\sqrt{4.1} \times 10 \times \sqrt{3} : \sqrt{3} : \sqrt{1}$